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Technical Comments

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Comment on "Cooling Fin Design"

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IN A recent paper, Bertola and Cafaro [1] posed a design problem for cooling fins whereby the temperature boundary condition at both ends (T_b at the base and T_t at the tip) and the heat flux at the base q_b were specified, and it was desired to determine the corresponding fin length L. Their analysis appeared to assume a free-ended tip, as indicated by Fig. 1 in their paper, and the rate of heat transfer at the tip was not considered. Their mathematical analysis based on one-dimensional, steady-state heat conduction led them to conclude that for fixed values of T_b and T_t, there is a range of q_b for which there are two possible solutions for L. Bertola and Cafaro attributed the existence of two possible solutions to the use of three conditions (T_b, T_t, and q_b) in solving the pertaining second-order differential equation. It is demonstrated in this Comment that the two solutions, while mathematically possible, correspond to two completely different physical arrangements and thus, they cannot be considered in design as alternative solutions to the same set of conditions. It is also shown that fins with prescribed tip temperature are typically connected to another body at the tip, except for a certain restrictive condition.

Consider a fin of uniform cross-sectional area A_c, perimeter P, length L, thermal conductivity k, base temperature T_b, and tip temperature T_t immersed in a medium at a temperature T_a with a uniform convection heat transfer coefficient h (as shown in Fig. 1). The governing energy-balance equation is given by [2–4]

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{1}$$

where x is the distance along the fin measured from the base (see Fig. 1), $\theta = T - T_a$, T is the fin temperature, and $m = \sqrt{hP/(kA_c)}$. The solutions for the temperature distribution and the rate of heat transfer at the base q_b are given by [2–4]

$$\frac{\theta}{\theta_b} = \frac{T - T_a}{T_b - T_a} = \frac{(\theta_t/\theta_b)\sinh(mx) + \sinh(m(L - x))}{\sinh(mL)} \tag{2}$$

and

$$q_b = kA_c m\theta_b(\cosh(mL) - \theta_t/\theta_b)/\sinh(mL) \tag{3}$$

The various possible types of temperature profiles that can be obtained from Eq. (2) are shown in Fig. 2. Profile A in Fig. 2 corresponds to a negative temperature gradient at the tip and consequently, a positive value of heat transfer rate at the tip q_t, while profile B corresponds to a zero temperature gradient at the tip ($q_t = 0$), and profile C corresponds to positive values of the tip temperature gradient and negative q_t. The value of q_t can be obtained easily from Eq. (2) using $q_t = -kA_c(dT/dx)_{x=L}$, resulting in

$$q_t = kA_c m\theta_b(1 - (\theta_t/\theta_b)\cosh(mL))/\sinh(mL) \tag{4}$$

By setting $q_t = 0$ in Eq. (4), we can obtain the condition for the type-B profile in Fig. 2 as

$$(\theta_t/\theta_b)\cosh(mL) = 1 \tag{5}$$

Therefore, type-A profiles correspond to $(\theta_t/\theta_b)\cosh(mL) < 1$, and type-C profiles correspond to $(\theta_t/\theta_b)\cosh(mL) > 1$.

The previous results for fins with a prescribed tip temperature suggest that the tip for cases B and C in Fig. 2 cannot be free ended and open to the surrounding ambient. Rather, the tip must be connected to another body that supplies heat into the fin (for case C), or provides insulation at the tip for case B. For type-A profiles, the fin may be free ended or connected to another body. The following condition must be satisfied for free-ended fins with prescribed temperature $-k(d\theta/dx)_{x=L} = h\theta_t$. Using Eq. (2), we get the following condition for free-ended fins:

$$\frac{\theta_t}{\theta_b} = \frac{1}{\cosh(mL) + \frac{hL/k}{mL}\sinh(mL)} \tag{6}$$

An example was provided by Bertola and Cafaro [1] to demonstrate the results of their analysis. The following parameters were specified in the example: $T_b = 373$ K, $T_t = 323$ K, $T_a = 293$ K, and $m = 1$. They plotted the rate of heat transfer at the base in terms of the parameter b [$=q_b/(kA_cT_a)$] versus mL and obtained the curve marked b in Fig. 3. Bertola and Cafaro then noted that there is one possible solution for mL if b is greater than 0.273, but for values of b between 0.253 and 0.273, there are two possible solutions for mL. To shed some light on these results, the rate of heat transfer at the tip was calculated by the present authors in terms of the parameter c [$=q_t/(kA_cT_a)$] and the results are shown in Fig. 3. It is clear from Fig. 3 that c is positive for $mL < 1.641$ (i.e., type-A

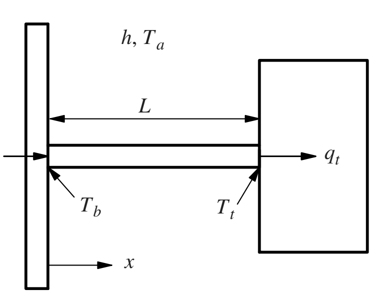

Fig. 1 Geometry and boundary conditions.

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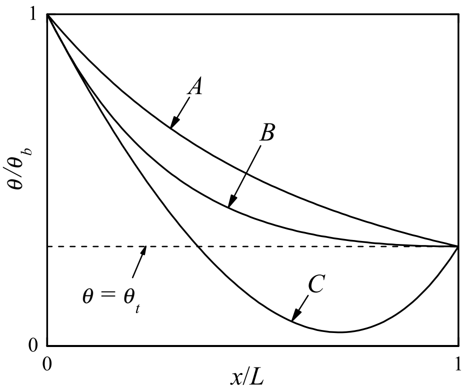

Fig. 2 Types of temperature profiles.

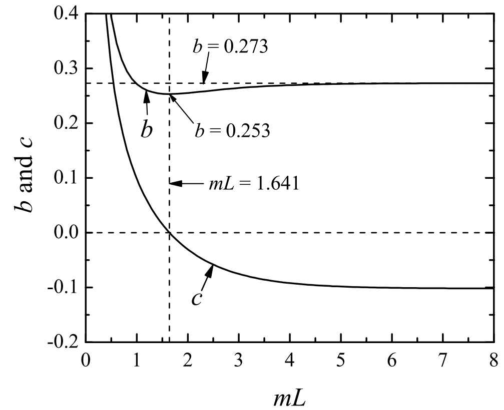

Fig. 3 Base and tip heat transfer rates.

temperature profiles), $c = 0$ at $mL = 1.641$ (i.e., type-B temperature profile), and c is negative for $mL > 1.641$ (type-C profiles). For $mL \geq 1.641$, the fin tip must be attached to a body providing heat input or insulation at the tip. For $mL < 1.641$, the tip is free ended and open to the ambient only if condition (6) is satisfied, otherwise, the tip must be attached to a body receiving heat from the fin. Thus, the solutions corresponding to $mL < 1.641$ are for a physical arrangement that is different from those corresponding to $mL > 1.641$, and cannot be regarded as two possible solutions for the same conditions.

To summarize, care must be exercised when using Eqs. (2) and (3) for the case of simple fins with a prescribed tip temperature. In most cases, the fin is connected to another body at the tip that maintains its temperature at the prescribed value. Only when condition (6) is satisfied would the tip be free ended and open to the ambient. Therefore, the heat transfer rate at the tip, given by Eq. (4), must be taken into consideration when analyzing the solutions obtained during fin design.

References

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